

Couplings of gravity to antisymmetric gauge fields

Xavier Bekaert^a, Bernard Knaepen^{a,b*}
and Christiane Schomblond^a

^a *Faculté des Sciences, Université Libre de Bruxelles,
Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium*

^b *DAMTP, Centre for Mathematical Sciences,
Wilberforce Road, CAMBRIDGE CB3 0WA, UK*

Abstract

We classify all the first-order vertices of gravity consistently coupled to a system of 2-form gauge fields by computing the local BRST cohomology $H(s|d)$ in ghost number 0 and form degree n . The consistent deformations are at most linear in the undifferentiated two-form, confirming the previous results of [1] that geometrical theories constructed from a nonsymmetric gravity theory are physically inconsistent or trivial. No assumption is made here on the degree of homogeneity in the derivatives nor on the form of the gravity action.

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1 Introduction

Long ago, the question has been addressed of constructing geometric field theories of gravity from the non-symmetric metric

$$G_{\mu\nu} \equiv g_{\mu\nu} + B_{\mu\nu} , \quad (1.1)$$

where $g_{\mu\nu}$ is the symmetric part of $G_{\mu\nu}$ and $B_{\mu\nu}$ is an antisymmetric tensor field. Such a model was first discussed in 1925 by Einstein [2] in an attempt to unify gravity and electromagnetism, and developed by himself and others [3]. In some sense, modern string theory has revived this attempt at geometric unification of forces [4]

But the couplings of a gauge field like a massless 2-form $B_{\mu\nu}$ and a (symmetric) metric $G_{\mu\nu}$ are severely restricted by consistency requirements, thereby disqualifying the old theory of Einstein and its modern versions (like the “nonsymmetric gravity theory” of Moffat [5]). The first proof of the physical inconsistency of these models has been given by T. Damour, S. Deser and J. McCarthy [1]. They have shown that all “geometric” actions (see [1] for a precise explanation of the sense of this term) homogeneous of order two in the number of derivatives violate standard physical requirements. In this paper, we confirm their results while relaxing some of the assumptions made by these authors.

To analyze the generic geometric models, they had to expand the action in powers of $B_{\mu\nu}$ about a classical symmetric background, and consider the merits of the resulting theory, that means study the consistency of the deformations of a theory of gravity coupled to a differential 2-form in terms of standard physical criteria: absence of negative-energy excitations and coherence of the degree-of-freedom content. The first point of their argument is that the absence of negative-energy excitations requires (whatever the gravitational background) the B expansion to begin with the (quadratic) kinetic term $H_{\mu\nu\lambda}H^{\mu\nu\lambda}$ where $H_{\lambda\mu\nu} \equiv \partial_\lambda B_{\mu\nu} + \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu}$ is the field strength. This action has the usual gauge invariance $\delta B_{\mu\nu} = \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu$. Secondly, geometric actions homogeneous in two derivatives generate powers of the undifferentiated $B_{\mu\nu}$. These higher-power terms generically violate the gauge invariance of the leading kinetic term. Furthermore, it is even impossible to overcome this problem by a deformation of the abelian gauge invariance [1].

A modern useful tool to study consistent deformations is the BRST field-antifield formalism. It is known that if s is the BRST operator of the free theory, the first order vertices of consistent deformations are constrained to belong to the cohomological group $H_0^n(s|d)$ of s modulo d in ghost number 0 and form degree n (the dimension of spacetime) [6]. By using the powerful tool of homological algebra, we completely determine all non trivial consistent deformations at first order in the coupling constant for a system of abelian 2-forms coupled to gravity for $n \geq 4$.¹

Our result is that all non trivial first-order consistent deformations are of the four following types:

- Strictly invariant polynomials in the curvatures $R_{\mu\nu\rho\sigma}$ and $H_{\mu\nu\rho}^A$.
- Couplings invariant under the abelian transformation up to a divergence. They generalize the purely gravitational deformations previously known in the literature (denoted as \mathcal{A}_{chiral} in [7]) and the Chern-Simons type self-coupling of the 2-forms (which exist only for specific values of n).
- Exotic couplings of the 2-form gauge fields to gravitational Chern-Simons forms (see [8]). These interactions deform the gauge transformations of the free theory in a non trivial way.

¹For $n < 4$, the 2-forms do not carry any degree of freedom, hence this case is not considered.

- The Freedman-Townsend coupling [9] (only allowed for $n = 4$), where the abelian gauge transformations for the 2-forms are non trivially deformed.

We notice that non trivial deformations of “gravity + 2-form” are at most linear in the undifferentiated $B_{\mu\nu}$ which confirms the results of Damour et al. that geometric actions including higher powers of the undifferentiated $B_{\mu\nu}$ are physically inconsistent or trivial. No assumption is made here on the degree of homogeneity in the derivatives nor on the form of the gravity action (except that it must define a normal theory, according to the terminology of [10]).

In the next section, we determine all the cohomological class of $H(s|d)$ in ghost number 0 and form degree n and we relate those classes to the first-order consistent deformations of the theory.

2 The model

Throughout the paper we shall work in the vielbein formulation of gravity. Denoting the vielbein fields by e_μ^a , we define the inverse vielbeins, the metric, the Christoffel connection and the spin connection through,

$$E_a^\mu e_\mu^b = \delta_b^a, \quad e_\mu^a E_a^\nu = \delta_\mu^\nu, \quad g_{\mu\nu} = e_{\mu a} e_{\nu}^a, \quad g^{\mu\nu} = E^{a\mu} E_a^\nu, \quad (2.1)$$

$$\partial_\mu e_\nu^a - \omega_{\mu b}^a e_\nu^b - \Gamma_{\mu\nu}^\rho e_\rho^a = 0, \quad \omega_\mu^{ab} = -\omega_\mu^{ba}, \quad \Gamma_{\mu\nu}^\rho = \Gamma_{\nu\mu}^\rho, \quad (2.2)$$

where Lorentz indices (a, b, \dots) are raised and lowered with the Minkowski metric.

As we pointed out in the introduction, our calculations are valid for a wide range of gravity action. For simplicity, the model we consider here is the standard Einstein-2-form system. Its lagrangian reads,

$$\mathcal{L}_0/e = \frac{1}{2}R - \frac{1}{12}g^{\mu\lambda}g^{\nu\sigma}g^{\rho\theta}H_{\mu\nu\rho}^A H_{A\lambda\sigma\theta}, \quad (2.3)$$

where e is the determinant of the vielbeins, $R = R_{\mu\nu}^{ab} E_a^\mu E_b^\nu$ is the Riemann curvature and $H_{\mu\nu\rho}^A$ are the 2-form field strengths.

Because of the gauge invariance of the classical action under the diffeomorphisms, local Lorentz transformations and gauge transformations of the 2-form potentials ($B_{\mu\nu}^A \rightarrow B_{\mu\nu}^A + \partial_\mu \epsilon_\nu^A - \partial_\nu \epsilon_\mu^A$), we are led to introduce within the field-antifield formalism the following set of ghosts: ξ^μ (diffeomorphisms ghosts), C^{ab} (Lorentz ghosts), B_μ^A, B^A (2-form ghosts). The extra ghosts B^A (“ghosts of ghosts”) required in the 2-form sector arise because the gauge transformations of the 2-form potentials are reducible. Indeed, they vanish when the gauge parameters are set to $\epsilon_\mu^A = \partial_\mu \lambda^A$ where λ^A are arbitrary functions. In the sequel we will denote by Φ^Γ all the fields and ghosts: $\{\Phi^\Gamma\} = \{e_\mu^a, B_{\mu\nu}^A, \xi^\mu, C^{ab}, B_\mu^A, B^A\}$. The corresponding antifields will be denoted by Φ_Γ^* : $\{\Phi_\Gamma^*\} = \{e_a^{*\mu}, B_A^{*\mu\nu}, \xi_\mu^*, C_{ab}^*, B_A^{*\mu}, B_A^*\}$. In terms of these variables the minimal solution of the BRST master equation reads:

$$\begin{aligned} S = \int d^n x (\mathcal{L}_0 - \xi^\mu \Phi_\Gamma^* \partial_\mu \Phi^\Gamma + e_a^{*\mu} (e_\nu^a \partial_\mu \xi^\nu + e_\mu^b C_b^a) + \frac{1}{2} C_K^* C^I C^J f_{IJ}^K \\ + B_A^{*\mu\nu} (\partial_\mu B_\nu^A + \partial_\mu \xi^\rho B_{\rho\nu}^A) - B_A^{*\mu} (\partial_\mu B^A + \partial_\mu \xi^\rho B_\rho^A)). \end{aligned} \quad (2.4)$$

In the above expression the f_{IJ}^K denote the structure constants of \mathcal{G}_L . We have also used the notation, $C^I = C^{ab}$. The action of the BRST differential on the variables is then defined as: $s\Phi^A = -\frac{\delta^R S}{\delta \Phi_A^*}$, $s\Phi_A^* = \frac{\delta^R S}{\delta \Phi^A}$.

In order to obtain the possible deformations of (2.3) we need to compute the local BRST cohomology $H(s|d)$ in the algebra of local forms. These are by definition linear combinations of spacetime forms $\omega^\tau(x, dx)$ with coefficients that are local functions. By local functions it is meant functions which depend polynomially on the variables $\partial_{\mu_1 \dots \mu_j} \Phi^\Gamma$ and $\partial_{\mu_1 \dots \mu_j} \Phi_\Gamma^*$ except the undifferentiated vielbeins e_μ^a for which a smooth and regular dependence in an open neighborhood of some regular background configuration ($\det(e_\mu^a) \neq 0$) is allowed. We shall denote the algebra of local forms by \mathcal{E} and the algebra of local functions by \mathcal{A} . Thus: $\mathcal{E} = \Omega(M) \otimes \mathcal{A}$; $a \in \mathcal{E} \Leftrightarrow a = \sum_\tau \omega^\tau \alpha_\tau$, $\omega^\tau \in \Omega(M)$, $\alpha_\tau \in \mathcal{A}$.

For gravitational theories it turns out that the local BRST cohomology in form degree n and ghost number g can easily be obtained from the cohomology $H(s)$ of the BRST differential s [11]. Indeed, one has the following isomorphism²:

$$H_g^n(s|d, \mathcal{E}) \simeq \frac{H^{g+n}(s, \mathcal{A})}{\mathbf{R}} \quad (2.5)$$

In particular, this isomorphism holds in ghost number 0 and therefore for the consistent deformations. It is implemented as follows: if α^n is a representative of $H^n(s, \mathcal{A})$, then the corresponding representative of $H_0^n(s|d, \mathcal{E})$ is given by $a_0^n = \frac{1}{n!} b^n \alpha^n$ where b is the operator defined by $b = dx^\mu \frac{\partial}{\partial \xi^\mu}$ [11].

The first part of the analysis of $H(s)$ consists in finding new generators of the algebra \mathcal{A} which isolate a contractible part of the algebra with respect to the differential s . Taking into account the results of [11, 12] we define the new basis of generators of \mathcal{A} as:

$$\{\mathcal{T}^r\} = \{D_{a_1} \dots D_{a_k} R_{ab}^I, D_{a_1} \dots D_{a_k} H_{bcd}^A : k = 0, 1, \dots\}, \quad (2.6)$$

$$\{\mathcal{T}_\Gamma^*\} = \{D_{a_1} \dots D_{a_k} \hat{\Phi}_\Gamma^* : k = 0, 1, \dots\}, \quad (2.7)$$

$$\hat{\xi}^a = \xi^\mu e_\mu^a, \hat{C}^I = C^I + \xi^\mu \omega_\mu^I, \quad (2.8)$$

$$\hat{B}^A = B^A + \xi^\mu B_\mu^A + \frac{1}{2} \xi^\nu \xi^\mu B_{\mu\nu}^A, \quad (2.9)$$

$$\{U_l\} = \{\partial_{(\mu_1 \dots \mu_k} e_{\mu)}^a, \partial_{(\mu_1 \dots \mu_k} \omega_{\mu)}^I, \partial_{(\mu_1 \dots \mu_k} B_{\nu_1)}^A, \partial_{(\mu_1 \dots \mu_k} B_{[\nu_1] \nu_2]}^A : k = 0, 1, \dots\}, \quad (2.10)$$

$$\{V_l\} = \{\partial_{(\mu_1 \dots \mu_k} s e_{\mu)}^a, \partial_{(\mu_1 \dots \mu_k} s \omega_{\mu)}^I, \partial_{(\mu_1 \dots \mu_k} s B_{\nu_1)}^A, \partial_{(\mu_1 \dots \mu_k} s B_{[\nu_1] \nu_2]}^A : k = 0, 1, \dots\}, \quad (2.11)$$

where,

$$H_{abc}^A = E_a^\mu E_b^\nu E_c^\rho H_{\mu\nu\rho}^A \quad (2.12)$$

$$R_{ab}^I = R_{ab}^{cd} = E_a^\mu E_b^\nu R_{\mu\nu}^{cd}, \quad (2.13)$$

$$\omega_\mu^I = \omega_\mu^{ab}, \quad (2.14)$$

$$\{\hat{\Phi}_\Gamma^*\} = \{\hat{e}_a^{*b}, \hat{B}_A^{*ab}, \hat{\xi}_a^*, \hat{C}_I^*, \hat{B}_A^{*a}, \hat{B}_A^*\}, \quad (2.15)$$

$$\hat{e}_a^{*b} = e_\mu^b e_a^{*\mu} / e, \hat{C}_I^* = C_I^* / e, \quad (2.16)$$

$$\hat{B}_A^{*ab} = e_\mu^a e_\nu^b B_A^{*\mu\nu} / e, \hat{B}_A^{*a} = -e_\mu^a B_A^{*\mu} / e, \hat{B}_A^* = B_A^* / e \quad (2.17)$$

$$\hat{\xi}_a^* = E_a^\mu (\xi_\mu^* - A_\mu^I C_I^* - B_{\mu\nu}^A B_A^{*\nu} - B_\mu^A B_A^*) / e \quad (2.18)$$

It is easily shown that any local function can be expressed in terms of the new generators.

²This isomorphism holds because we assume the spacetime manifold to be homeomorphic to \mathbf{R}^n .

	parity	antigh	puregh	gh
s	1	-	-	1
δ	1	-1	0	1
γ	1	0	1	1
\mathcal{T}^r	0	0	0	0
$\hat{\xi}^a$	1	0	1	1
\hat{C}^I	1	0	1	1
\hat{B}^A	0	0	2	2
$\mathcal{T}_{\bar{r}}^*$	1	1	0	-1
\hat{e}_a^{*b}	1	1	0	-1
\hat{B}_A^{*ab}	1	1	0	-1
$\hat{\xi}_a^*$	0	2	0	-2
\hat{C}_I^*	0	2	0	-2
\hat{B}_A^{*a}	0	2	0	-2
\hat{B}_A^*	1	3	0	-3

The first advantage of the new basis is that on all but the variables U_l and V_l , the action of s takes the familiar form:

$$s = \delta + \gamma, \quad (2.19)$$

with,

$$\delta \mathcal{T}^r = \delta \hat{\xi}^a = \delta \hat{C}^I = \delta \hat{B}^A = 0, \quad (2.20)$$

$$\delta \hat{e}_a^{*b} = -(R_a^b - \frac{1}{2} \delta_a^b R) - \frac{1}{2} H_{acd}^A H_A^{bcd} - \frac{1}{12} \delta_a^b H_{cde}^A H_A^{cde}, \quad (2.21)$$

$$\delta \hat{C}_{ab}^* = -2 \hat{e}_{[ab]}^*, \delta \hat{\xi}_a^* = -D_b \hat{e}_a^{*b} + \frac{1}{p_A!} \hat{B}_A^{*b_1 b_2} \hat{H}_{ab_1 b_2}^A \quad (2.22)$$

$$\delta \hat{B}_A^{*a_1 a_2} = D_b \hat{H}_A^{ba_1 a_2}, \delta \hat{B}_A^{*a_1} = D_b \hat{B}_A^{*ba_1}, \delta \hat{B}_A^* = D_b \hat{B}_A^{*b}, \quad (2.23)$$

$$\delta D_{a_1} \dots D_{a_k} \hat{\Phi}_\Gamma^* = D_{a_1} \dots D_{a_k} \delta \hat{\Phi}_\Gamma^*, \quad (2.24)$$

$$\gamma \mathcal{T}^r = (\hat{\xi}^a D_a + \hat{C}^I \delta_I) \mathcal{T}, \quad \gamma \mathcal{T}_{\bar{r}}^* = (\hat{\xi}^a D_a + \hat{C}^I \delta_I) \mathcal{T}_{\bar{r}}^* \quad (2.25)$$

$$\gamma \hat{\xi}^a = \hat{C}_b^a \hat{\xi}^b, \quad \gamma \hat{C}^I = \frac{1}{2} \hat{C}^J \hat{C}^K f_{KJ}^I + \hat{F}^I, \quad \gamma \hat{B}^A = \hat{H}^A, \quad (2.26)$$

where

$$\hat{R}^I = \frac{1}{2} \hat{\xi}^c \hat{\xi}^d R_{cd}^I, \quad \hat{H}^A = \frac{1}{6} \hat{\xi}^c \hat{\xi}^b \hat{\xi}^a H_{abc}^A. \quad (2.27)$$

In the above equations, the notation $\hat{C}^I \delta_I \mathcal{T}^r$ stands for the Lorentz infinitesimal transformation of \mathcal{T}^r with the infinitesimal parameter replaced by the ghost \hat{C}^I . For example, if $\mathcal{T}^r = \mathcal{T}^a$ is a contravariant vector then $\hat{C}^I \delta_I \mathcal{T}^a = \hat{C}_b^a \mathcal{T}^b$.

The gradings associated to δ and γ are respectively the *antighost* number (denoted *antigh*) and the *pureghost* number (denoted *puregh*); their sum is equal to the *ghost* number (denoted *gh*). The table summarizes the various gradings associated to the operators, the fields, the ghosts and the antifields.

The second advantage of the new basis is that it exhibits a manifestly contractible part of the algebra \mathcal{A} . Indeed, by construction, the variables U_l and V_l are mapped on each other by the BRST differential,

$$sU_l = V_l, \quad sV_l = 0, \quad (2.28)$$

and since the BRST differential does not mix the U_l and V_l with the rest of the variables, each couple $\{U_l, V_l\}$ drops out from the cohomology and we have $H(s, \mathcal{A}) \simeq H(s, \mathcal{A}_2)$ where \mathcal{A}_2 is the algebra generated by the set $\{\mathcal{T}^r, \mathcal{T}_r^*, \hat{\xi}^a, \hat{C}^I, \hat{B}^A\}$.

To summarize, the above discussion indicates that to obtain $H_0^n(s|d, \mathcal{E})$ we only need to calculate $H^n(s, \mathcal{A}_2)$. This is the subject of the next section.

3 BRST cohomology in \mathcal{A}_2 and consistent vertices

In order to get the cohomology $H^n(s, \mathcal{A}_2)$ we need to solve the equation,

$$s\alpha^n = 0, \quad \alpha^n \in \mathcal{A}_2, \quad (3.1)$$

where two solutions of (3.1) are identified if they differ by an s -exact contribution, i.e. $\alpha^n \sim \alpha^n + s\beta^{n-1}$ with $\beta^{n-1} \in \mathcal{A}_2$.

The approach we follow is identical to the one developed in [11] so we only emphasize here the main ideas and the new results. A more in-depth presentation will be given in [13].

First, the cocycle α^n is decomposed according to a degree called the $\hat{\xi}^a$ -degree which counts the polynomial degree in the variables $\hat{\xi}^a$,

$$\alpha^n = \sum_{k=0}^n \alpha_k. \quad (3.2)$$

According to the $\hat{\xi}^a$ -degree, the BRST differential decomposes into four parts, $s = s_0 + s_1 + s_2 + s_3$, which can be read off from (2.20)-(2.26). The first term is given by,

$$s_0 = \delta + \gamma_L \quad (3.3)$$

where δ is the Koszul-Tate differential and γ_L is the longitudinal exterior derivative along the gauge orbits of \mathcal{G}_L ,

$$\gamma_L = -\frac{1}{2}\hat{C}^J\hat{C}^K f_{JK}^I \frac{\partial}{\partial \hat{C}^I} + \hat{C}^I \delta_I, \quad (3.4)$$

with,

$$\delta_{ab}\hat{\xi}_c = \eta_{bc}\hat{\xi}_a - \eta_{ac}\hat{\xi}_b, \quad \delta_I \hat{C}^J = -f_{IK}^J \hat{C}^K. \quad (3.5)$$

s_1 plays the rôle of an exterior covariant derivative whose differentials are the $\hat{\xi}^a$. Its action is given by,

$$s_1 \mathcal{T}^r = \hat{\xi}^a D_a \mathcal{T}^r, \quad s_1 \mathcal{T}_r^* = \hat{\xi}^a D_a \mathcal{T}_r^*, \quad s_1 \hat{\xi}^a = s_1 \hat{C}^I = s_1 \hat{B}^A = 0. \quad (3.6)$$

Finally, the operators s_2 and s_3 are given by,

$$s_2 = \hat{R}^I \frac{\partial}{\partial \hat{C}^I}, \quad s_3 = \hat{H}^A \frac{\partial}{\partial \hat{B}^A}. \quad (3.7)$$

According to the $\hat{\xi}^a$ -degree, eq. (3.1) decomposes into the following tower of equations:

$$0 = s_0 \alpha_l, \quad (3.8)$$

$$0 = s_0 \alpha_{l+1} + s_1 \alpha_l, \quad (3.9)$$

$$0 = s_0 \alpha_{l+2} + s_1 \alpha_{l+1} + s_2 \alpha_l, \quad (3.10)$$

\vdots

Up to trivial terms which only modify components of higher $\hat{\xi}^a$ -degree, eq. (3.8) indicates that α_l is an element of the cohomology $H(s_0, \mathcal{A}_2)$. Given the definition of s_0 , this cohomology is analyzed in a very similar fashion as the standard BRST cohomology for non-gravitational theories. Using the acyclicity of δ in antighost number $k > 0$, one can show that each class of $H(s_0, \mathcal{A}_2)$ admits an antifield independent representative so that the cocycle condition becomes, $\gamma_L \alpha_l = 0$. This equation is the well known coboundary condition for the Lie algebra cohomology of \mathcal{G}_L in a \mathcal{G}_L -module so the most general form for the non-trivial α_l is,

$$\alpha_l = \alpha_l^i(\hat{\xi}^a, \mathcal{T}^r) \omega_i(\theta_K(\hat{C}^I), \hat{B}^A), \quad \delta_I \alpha_l^i(\hat{\xi}^a, \mathcal{T}^r) = 0. \quad (3.11)$$

In (3.11), the ω_i are polynomials in the \hat{B}^A and the $\theta_K(\hat{C}^I)$ which are the primitive elements of the Lorentz Lie algebra cohomology. In $n = 2r$ and $n = 2r + 1$ dimensions, they are given by,

$$\theta_K(C) = C_{a_1}^{a_2} D_{a_2}^{a_3} \dots D_{a_{2K}}^{a_1}, \quad K = 1, 2, \dots, r-1, \quad (3.12)$$

$$\theta_r(C) = \begin{cases} C_{a_1}^{a_2} D_{a_2}^{a_3} \dots D_{a_{2r}}^{a_1} & \text{for } n = 2r + 1 \\ \epsilon_{a_1 b_1 \dots a_r b_r} C^{a_1 b_1} D^{a_2 b_2} \dots D^{a_r b_r} & \text{for } n = 2r \end{cases} \quad (3.13)$$

where $D_a^b = C_a^c C_c^b$.

Let us first consider the case $l = n$. Since we are interested in solutions of (3.1) in ghost number n , in (3.11) we necessarily have $\omega_i(\theta_K(\hat{C}^I), \hat{B}^A) = k_i$ where the k_i are constants (the $\hat{\xi}^a$ are of ghost number 1). In that case, no further condition is imposed on α_n and we have,

$$\alpha = \alpha_n = L(\mathcal{T}) \hat{\Theta}, \quad \delta_I L = 0, \quad (3.14)$$

where $\hat{\Theta} = \hat{\xi}^0 \dots \hat{\xi}^{n-1}$. The above elements of $H(s)$ give rise in $H_0^n(s|d, \mathcal{E})$ to the cocycles,

$$\alpha = L(\mathcal{T}) d^n x, \quad \delta_I L = 0, \quad (3.15)$$

which constitute the first set of consistent interactions announced in the introduction. Since these vertices are strictly gauge invariant they do not require any modification in the gauge transformations of the theory.

We now turn our attention to the case $l < n$. To that end, we substitute the general form (3.11) into eq. (3.9). By doing so, one easily proves [11] that $\alpha_l^i(\hat{\xi}^a, \mathcal{T}^r)$ has to obey the following equation,

$$s_1 \alpha_l^i + \delta \alpha_{l+1,1}^i = 0, \quad (3.16)$$

where $\alpha_{l+1,1}^i = \alpha_{l+1,1}^i(\hat{\xi}^a, \mathcal{T}^r, \mathcal{T}_{\overline{r}}^*)$ is of antighost number 1 and satisfies $\delta_I \alpha_{l+1,1}^i(\hat{\xi}^a, \mathcal{T}^r, \mathcal{T}_{\overline{r}}^*) = 0$. Trivial solutions of (3.16) ($\alpha_l^i = s_1 \beta_{l-1}^i + \delta \beta_l^i$) are irrelevant since they amount to trivial contributions in α .

Eq. (3.16) along with its coboundary condition define the so-called ‘‘invariant characteristic cohomology’’ $H_{char}^{inv}(d)$ (with d formally substituted by s_1) which plays a central rôle in the analysis

of any local field theory. Theorems concerning $H_{char}^{inv}(d)$ in the cases of pure gravity and p -form gauge theory can be found respectively in [10] and [14]. The extension of those results in the case of gravity coupled to a system of 2-forms is fully treated in [13]. Here, we may restrict our attention to solutions of eq. (3.16) in ghost number $\leq n-2$. Indeed, taking into account (3.12) and (3.13) we see that the $\theta_K(\hat{C}^I)$ and \hat{B}^A are at least of ghost number 2; therefore, to construct solutions of (3.1) in ghost number n , we necessarily have $l \leq n-2$. In that case, the most general solution of (3.16) is up to trivial terms given by [13],

$$\alpha_l^i = P^i(f_K, \hat{H}^A) + \delta_l^{n-3} k_A^i \hat{H}^A + \delta_l^{n-2} \delta_4^n k_{AB}^i \hat{H}^A \hat{H}^B. \quad (3.17)$$

In (3.17), the k_A^i and $k_{AB}^i = -k_{BA}^i$ are constants, \hat{H}^A is the Hodge dual of \hat{H}^A and the f_K are generators for the the \mathcal{G}_L -invariant polynomials in the \hat{R}^I . The f_K are given in the cases $n = 2r$ and $n = 2r + 1$ by:

$$f_K = \hat{R}_{a_1}^{a_2} \hat{R}_{a_2}^{a_3} \dots \hat{R}_{a_{2K}}^{a_1}, \quad K = 1, 2, \dots, r-1, \quad (3.18)$$

$$f_r = \begin{cases} \hat{R}_{a_1}^{a_2} \hat{R}_{a_2}^{a_3} \dots \hat{R}_{a_{2r}}^{a_1} & \text{for } n = 2r + 1 \\ \epsilon_{a_1 b_1 \dots a_r b_r} \hat{R}^{a_1 b_1} \dots \hat{R}^{a_r b_r} & \text{for } n = 2r. \end{cases} \quad (3.19)$$

From (3.17) we see that α_l^i involves two kinds of contributions. The first consists of the polynomials $P^i(f_K, \hat{H}^A)$ which obey (3.16) without the need for a term $\alpha_{l+1,1}^i$. One says that the corresponding α_l^i are *strongly* s_1 -closed. On the other hand, the last two terms of (3.17) are only *weakly* s_1 -closed since they require a term $\alpha_{l+1,1}^i$ in order to satisfy eq. (3.16).

Let us first consider the BRST cocycles generated by the strongly s_1 -closed α_l^i . Their part of lowest $\hat{\xi}^a$ -degree is of the form,

$$\alpha_l = P^i(f_K, \hat{H}^A) \omega_i(\theta_K(\hat{C}^I), \hat{B}^A). \quad (3.20)$$

We need to complete these α_l by terms of higher $\hat{\xi}^a$ -degree in order to obtain BRST cocycles. To do this, we associate to the θ_K the following quantities [11]:

$$q_K = \sum_{k=0}^{2K-1} (-)^k \frac{(2K)!(2K-1)!}{(2K+k)!(2K-k-1)!} \text{Str}(\hat{C} \hat{D}^k \hat{R}^{2K-k-1}), \quad (3.21)$$

with,

$$\hat{C} = \hat{C}^I T_I, \quad \hat{D} = \hat{C}^2, \quad \hat{R} = \hat{R}^I T_I. \quad (3.22)$$

In (3.22), the T_I are the matrices of the adjoint representation of $so(n-1, 1)$ except for $n = 2r$ in which the spinor representation is used. At lowest $\hat{\xi}^a$ -degree the q_K begin with θ_K and are such that $sq_K = f_K$.

The usefulness of the q_K stems from the fact that if α is a cocycle with α_l as in (3.20), then one may assume that $\alpha = \alpha(f_K, q_K, \hat{H}^A, \hat{B}^A) = P^i(f_K, \hat{H}^A) \omega_i(q_K, \hat{B}^A)$, i.e., α is obtained from α_l by replacing the θ_K by the q_K [13]. Furthermore, one can show that if $\alpha = \alpha(f_K, q_K, \hat{H}^A, \hat{B}^A) = s\beta$ then $\beta = \beta(f_K, q_K, \hat{H}^A, \hat{B}^A)$. The analysis of the BRST cocycles arising from (3.20) is therefore reduced to the investigation of the BRST cohomology in the so-called small algebra generated by the variable $q_K, f_K, \hat{H}^A, \hat{B}^A$.

This problem has been studied in detail for the pure gravitational case in [7]. It is shown that in ghost number n the BRST cocycles in the small algebra are of the ‘‘Chern-Simons’’ form,

$\alpha = q_K P(f_K)$. Here, the only difference comes from the presence of \hat{H}^A and \hat{B}^A among the generators of the small algebra; they are related by $s\hat{B}^A = \hat{H}^A$. The procedure is nearly identical to [7] and one can show [13] that in ghost number n the BRST cocycles are again of the Chern-Simons form, i.e.,

$$\alpha = q_K P_K(f_L, \hat{H}^A) + \hat{B}^A Q_A(f_K, \hat{H}^B). \quad (3.23)$$

The corresponding consistent deformations are obtained from (3.23) by the substitution $\hat{\xi}^a \rightarrow dx^\mu$. They constitute the second set of vertices announced in the introduction. Note that they are invariant up to a boundary term and therefore do not require a modification of the gauge transformations.

We now consider the BRST cocycles generated by the weakly s_1 -closed α_l^i . Taking into account (3.12), (3.13) and (3.17), we see that in spacetime dimension $n > 4$, the only possibility we have in ghost number n for α_l is,

$$\alpha_{n-3} = k_A^1 \overline{\hat{H}}^A \theta_1, \quad (3.24)$$

with $\theta_1 = C_a^b C_b^c C_c^a$. α_{n-3} has to be completed with terms of higher $\hat{\xi}^a$ -degree in order to obtain a BRST cocycle. To that end we introduce the following notation [14]:

$$q^{*A} = \overline{\hat{H}}^A + \overline{\hat{B}}_1^{*A} + \overline{\hat{B}}_2^{*A} + \overline{\hat{B}}_3^{*A}, \quad (3.25)$$

where the $\overline{\hat{B}}_j^{*A}$ are defined through, $\delta \overline{\hat{B}}_1^{*A} + d\overline{\hat{H}}^A = 0$, $\delta \overline{\hat{B}}_2^{*A} + d\overline{\hat{B}}_1^{*A} = 0$, $\delta \overline{\hat{B}}_3^{*A} + d\overline{\hat{B}}_2^{*A} = 0$ ($\overline{\hat{B}}_j^{*A}$ is proportional to the dual of the antifield of antighost j , with the $\hat{\xi}^a$ as differentials).

Using the above definition, it is straightforward to complete (3.24) to a BRST cocycle. Indeed, as can be seen by direct substitution,

$$\alpha = k_A^1 q^{*A} q_1 \quad (3.26)$$

$$= k_A^1 \overline{\hat{H}}^A \theta_1 + \alpha_{n-2} + \alpha_{n-1} + \alpha_n. \quad (3.27)$$

is a solution of (3.24). The corresponding elements of $H(s|d, \mathcal{E})$ are again obtained from (3.26) by the substitution $\hat{\xi}^a \rightarrow dx^\mu$. Their components of antighost number zero give rise to the following class of consistent vertices:

$$V = k_A^1 \overline{H}^A \text{Tr}(\omega R - \frac{1}{3} \omega^3) \quad (3.28)$$

where \overline{H}^A is the dual of the field strength H^A , $\omega = \omega_\mu^I dx^\mu T_I$ and $R = \frac{1}{2} dx^\mu dx^\nu R_{\mu\nu}^I T_I$ (the T_I are the matrices of the adjoint representation of $so(n-1, 1)$). These vertices require a modification in the gauge transformations since they are not invariant under the original ones. They belong to the third type of interactions described in the introduction.

In the particular case of spacetime dimension $n = 4$ further couplings are possible. Indeed, in ghost number $n = 4$ we have the following candidates for α_l ,

$$\alpha_l = k_A^1 \overline{\hat{H}}^A \theta_1 + k_A^2 \overline{\hat{H}}^A \theta_2 + k_{ABC} \overline{\hat{H}}^A \overline{\hat{H}}^B \hat{B}^C, \quad (3.29)$$

where $k_{ABC} = -k_{BAC}$ and $\theta_2 = \epsilon_{abcd} C^{ab} C^{ce} C_e^d$. The above α_l are easily completed to the following BRST-cocycles,

$$\alpha = k_A^1 q^{*A} q_1 + k_A^2 q^{*A} q_2 + k_{ABC} q^{*A} q^{*B} \hat{B}^A. \quad (3.30)$$

The first two terms yield consistent vertices identical to (3.28) (with the trace taken once in the adjoint representation and once in the spinorial representation of $so(n-1,1)$):

$$V = k_A^1 \overline{H}^A Tr_{adj}(\omega R - \frac{1}{3}\omega^3) + k_A^2 \overline{H}^A Tr_{sp}(\omega R - \frac{1}{3}\omega^3) \quad (3.31)$$

The last term in (3.30) produces the Freedman-Townsend coupling,

$$V = k_{ABC} \overline{H}^A \overline{H}^B B^C. \quad (3.32)$$

This vertex is again not invariant under the original gauge transformations and a modification of these transformations is imposed.

4 Comments

In this article we have computed the local BRST cohomology $H(s|d)$ in ghost number 0 and form degree n in order to obtain all the first-order vertices of gravity coupled to a system of 2-form gauge fields.

The first two types of couplings we obtain (strictly invariant polynomials in the curvatures and Chern-Simons forms) are consistent to all orders in the coupling constant since they are gauge invariant (up to a total derivative for the Chern-Simons forms) under the original gauge transformations. These vertices correspond to antifield independent representative of the BRST cohomology.

The last two types of vertices we describe are not invariant under the original gauge transformations since they depend non-trivially on the antifields. In that case a modification of the gauge transformations is required as well as the addition of higher-order vertices in order to obtain a theory consistent to all orders in the coupling constant. For the exotic couplings of the 2-form gauge fields to gravitational Chern-Simons forms, the full theory is of the Chapline-Manton type in which the 2-form curvature present in (2.3) is replaced by $H'_A = H_A + k_A Tr(\omega F - \frac{1}{3}\omega^3)$. The Freedman-Townsend coupling gives rise to a non-polynomial full theory. Polynomiality can be restored by describing the theory with a “first-order formulation” and the full lagrangian is simply a covariantized version of the original Freedman-Townsend theory.

Our analysis can be extended to cover the couplings of gravity to p -forms [13].

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